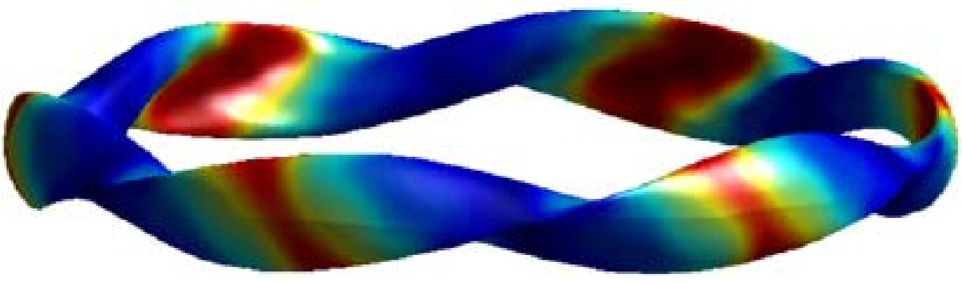
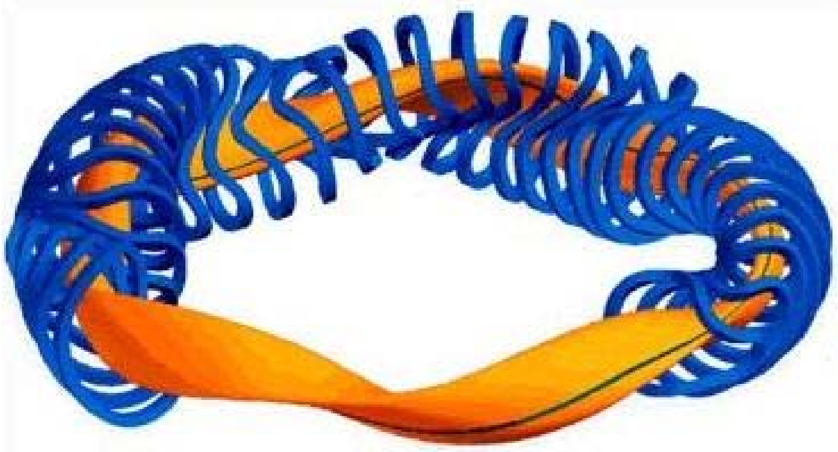
Omnigenity as generalized quasisymmetry in stellarators





Matt Landreman

Thanks to Peter Catto & Per Helander

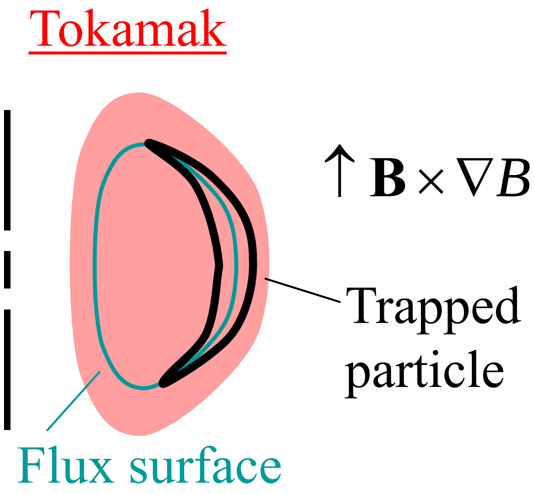
Landreman & Catto, Phys. Plasmas 19, 056103 (2012)

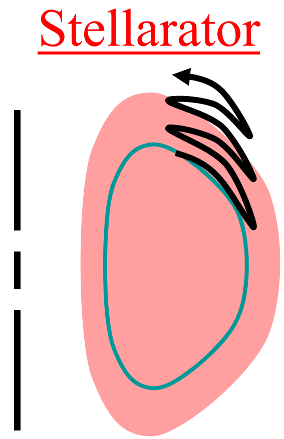
Supported by U.S. D.o.E. & ORISE

Preview

- “Omnigenity” = “collisionless particle trajectories are confined.”
- Quasisymmetry is sufficient but not necessary for omnigenity.
- Several properties of quasisymmetric plasmas apply with only minor modification to the larger set of omnigenous fields:
  - Have a “helicity” (M, N), like quasisymmetry.
  - Formulae for current & flow simplify dramatically.
- But, the radial electric field is different in quasisymmetric vs. omnigenous plasmas.

Omnigenity = no unconfined orbits.





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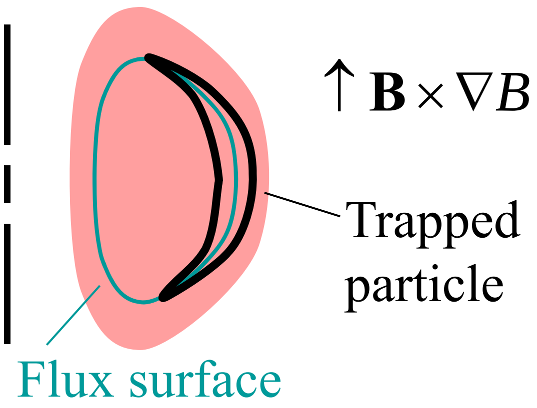
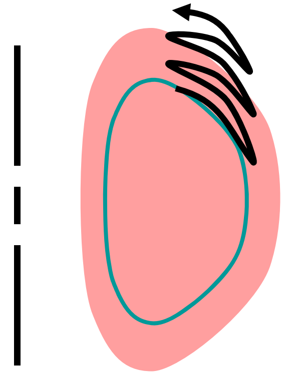
Tokamak



Stellarator



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For a reactor, then, a stellarator must be nearly *omnigenous*:

$$0 = \Delta\psi \text{ per bounce} = \oint_{\text{bounce}} \left(\mathbf{v}_d \cdot \nabla\psi\right) dt \qquad \text{for all } \mu \text{ and all trapped particles.}$$

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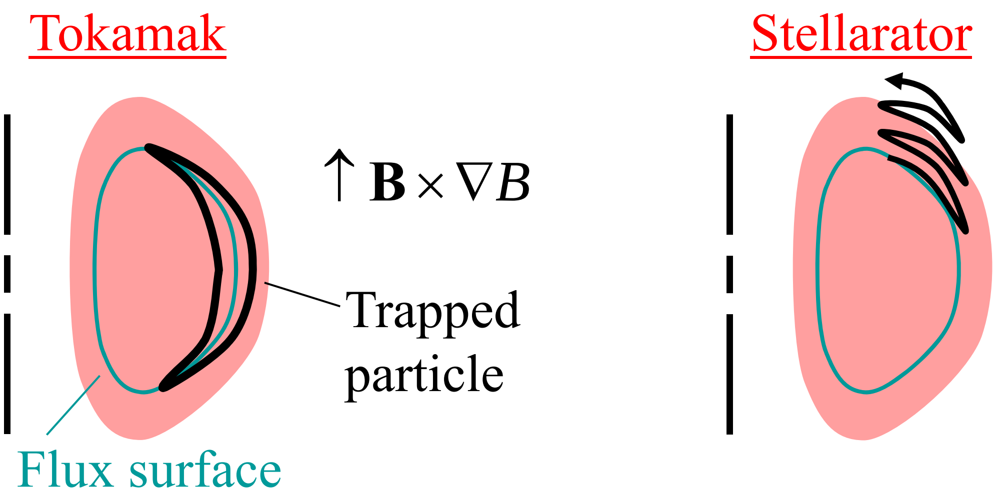

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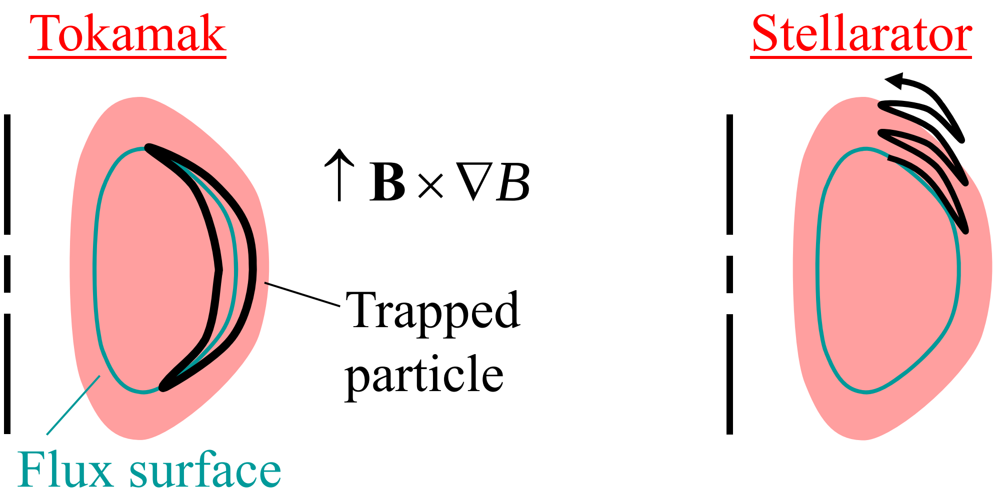

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Also equivalent: "effective helical ripple" $\varepsilon_{\text{eff}} \to 0$.

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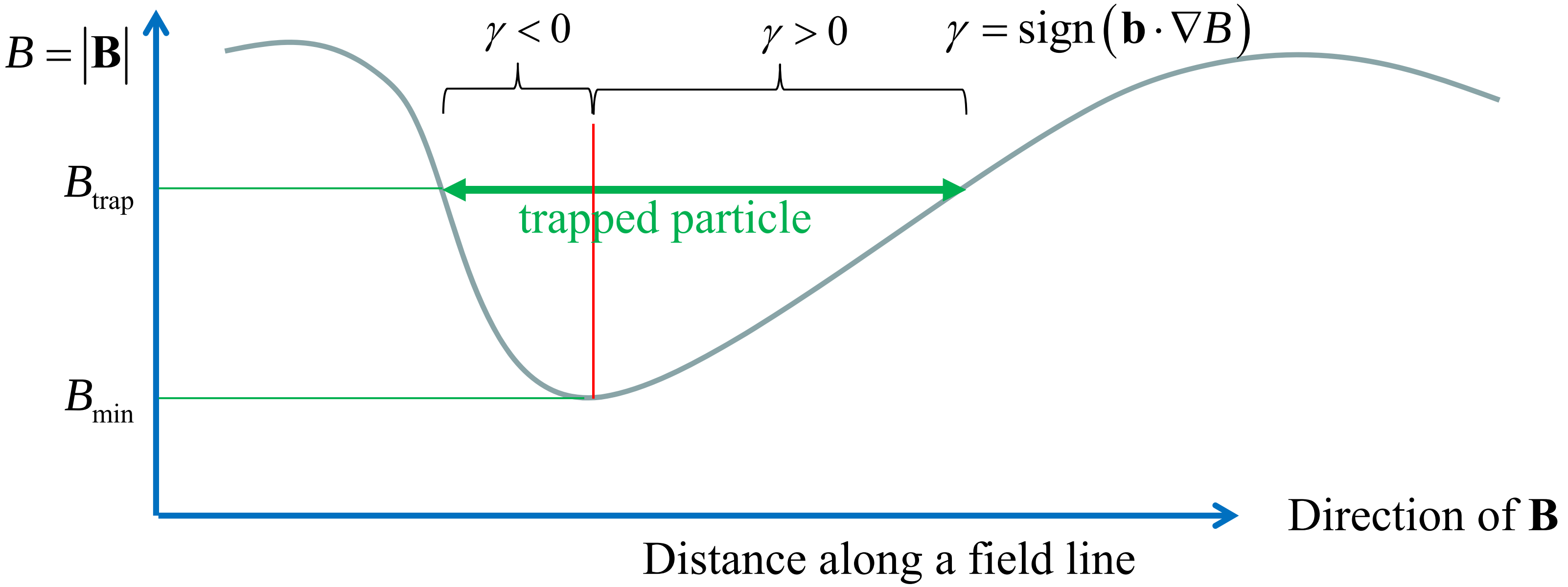

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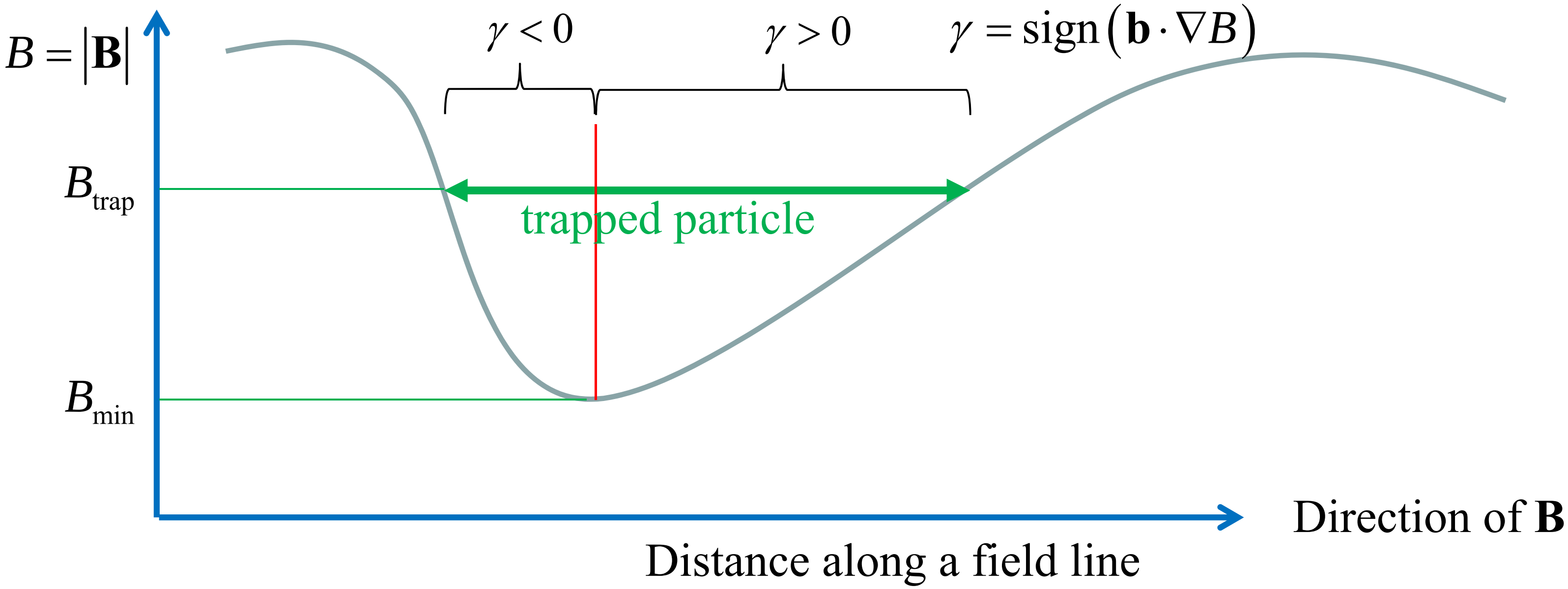

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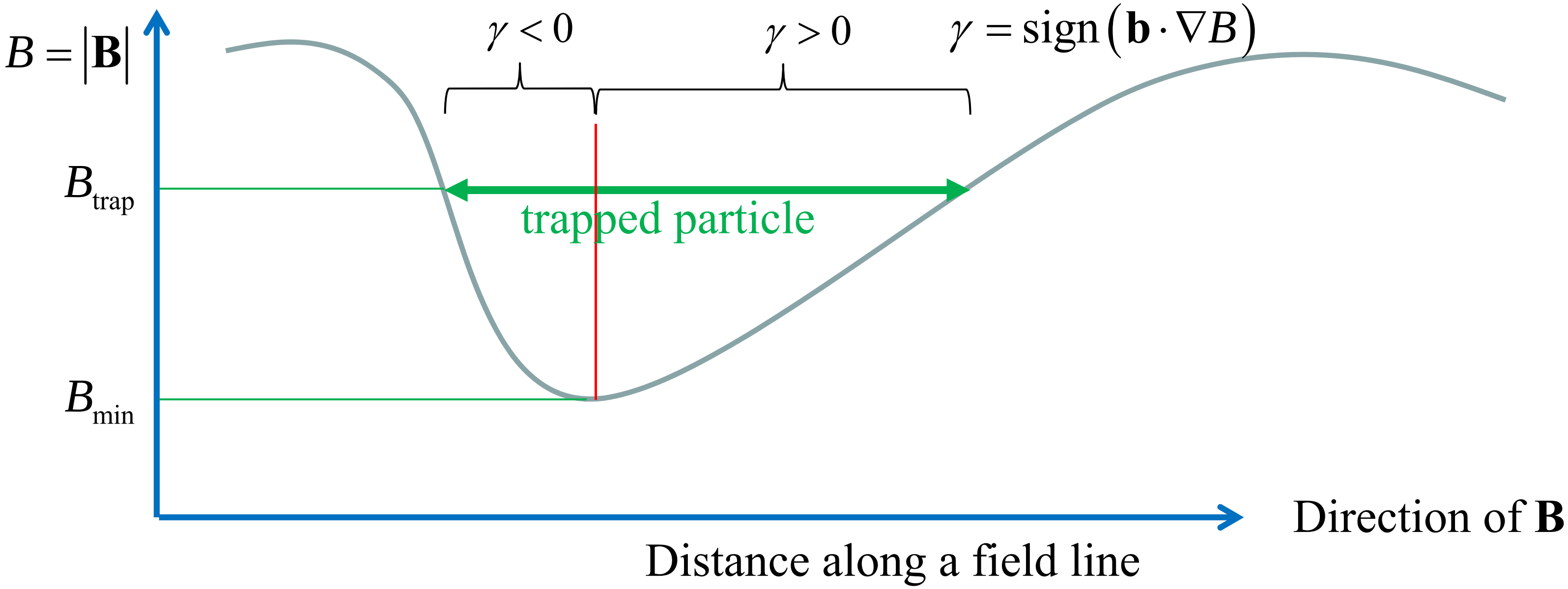
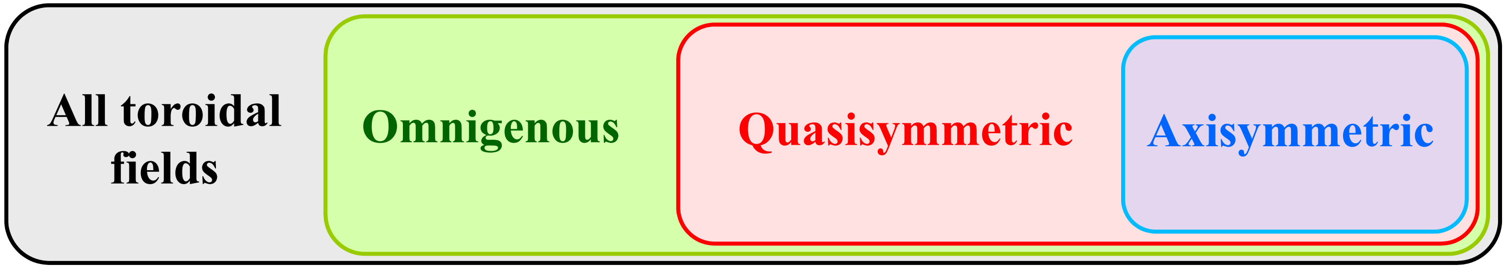


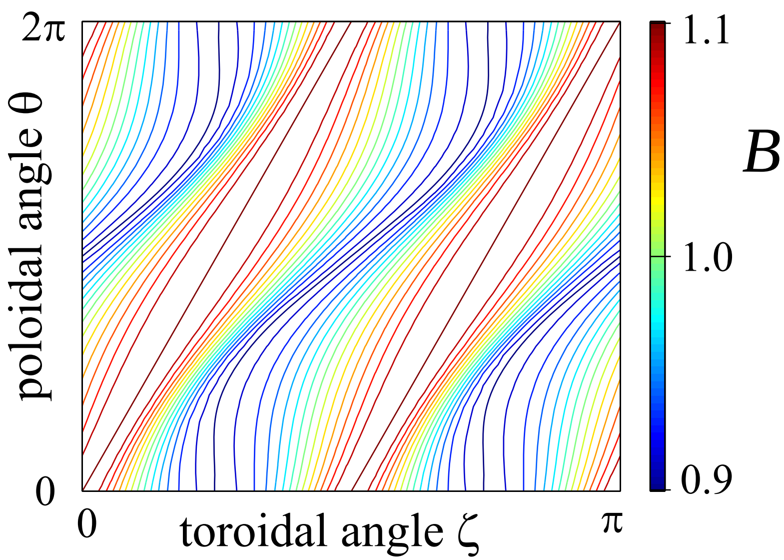
Quasisymmetry: $\dfrac{\mathbf{B}\times\nabla B\cdot\nabla\psi}{\mathbf{B}\cdot\nabla B}$ is a flux function, so integral is independent of γ, so $\sum_\gamma \gamma = 0$.

Omnigenity is more general than quasisymmetry.

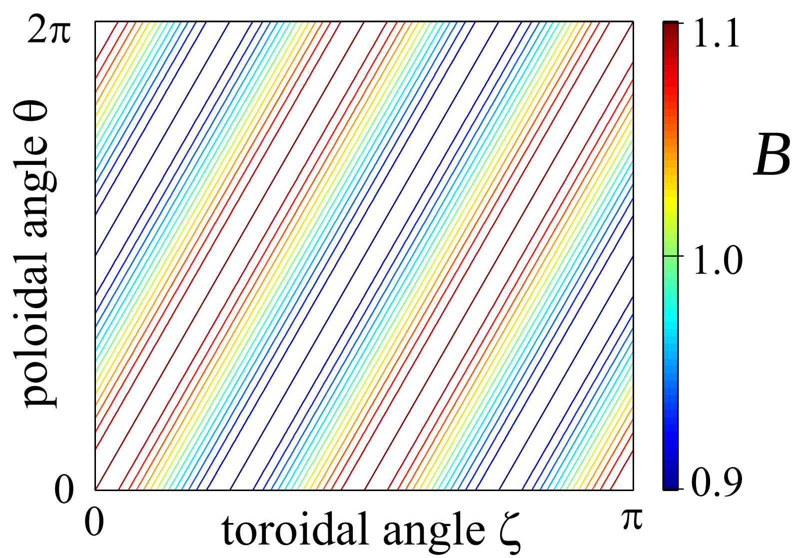
Cary & Shasharina, *PoP* (1997), *PRL* (1997)



Omnigenity: B contours may be curved



Quasisymmetry: $B = B(M\theta - N\zeta)$



Does a quasisymmetric stellarator permit larger flows than a general stellarator?

Usual ordering for mean flow **V** in kinetic theory: $\mathbf{V} \sim O(\rho_* v_{\mathrm{th,i}})$.
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In order for **V** to be as large as $\sim O(v_{\mathrm{th,i}})$, the $O(f \, v_{\mathrm{th,i}}/L)$ terms in the ion kinetic equation imply $\nabla B \times \nabla \psi \cdot \nabla \left(\mathbf{B} \cdot \nabla B \right) = 0$ (quasisymmetry).

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Actually, even Helander's solution cannot satisfy

$$mn\mathbf{V} \cdot \nabla \mathbf{V} = -\nabla p + ne\mathbf{E} + ne\mathbf{V} \times \mathbf{B}$$

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Simakov & Helander, *Plasma Phys. Control. Fusion* **53**, 024005 (2011):

In a nonaxisymmetric plasma, even if B is quasisymmetric, $\mathbf{V}\cdot\nabla\mathbf{V}$ drives a ϕ that is not.

$\Rightarrow$ Helically electrostatically trapped particles slow the plasma.

What does an omnigenous $B(\theta,\zeta)$ look like?

$$\oint \left(\mathbf{v}_d \cdot \nabla \psi \right) dt = 0$$

Determined by $B=|\mathbf{B}|$ on a flux surface

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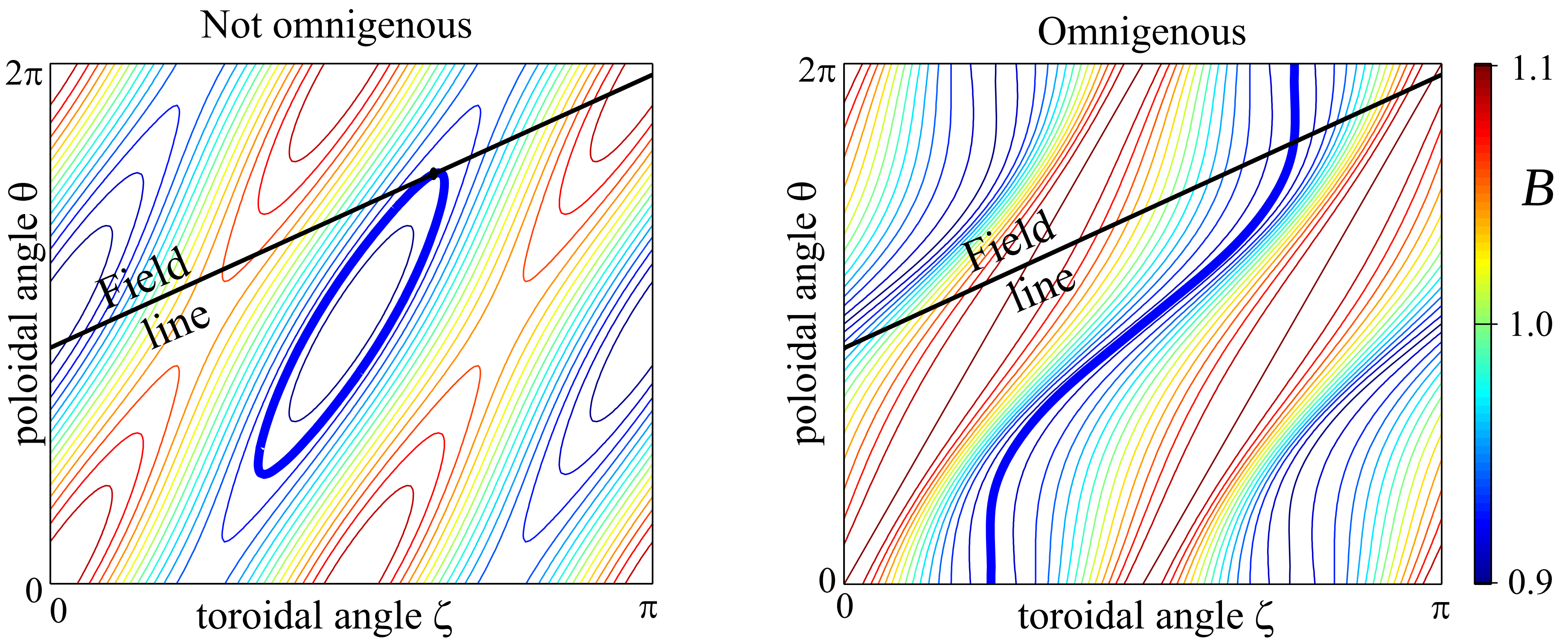
Contours of *B* on a flux surface:



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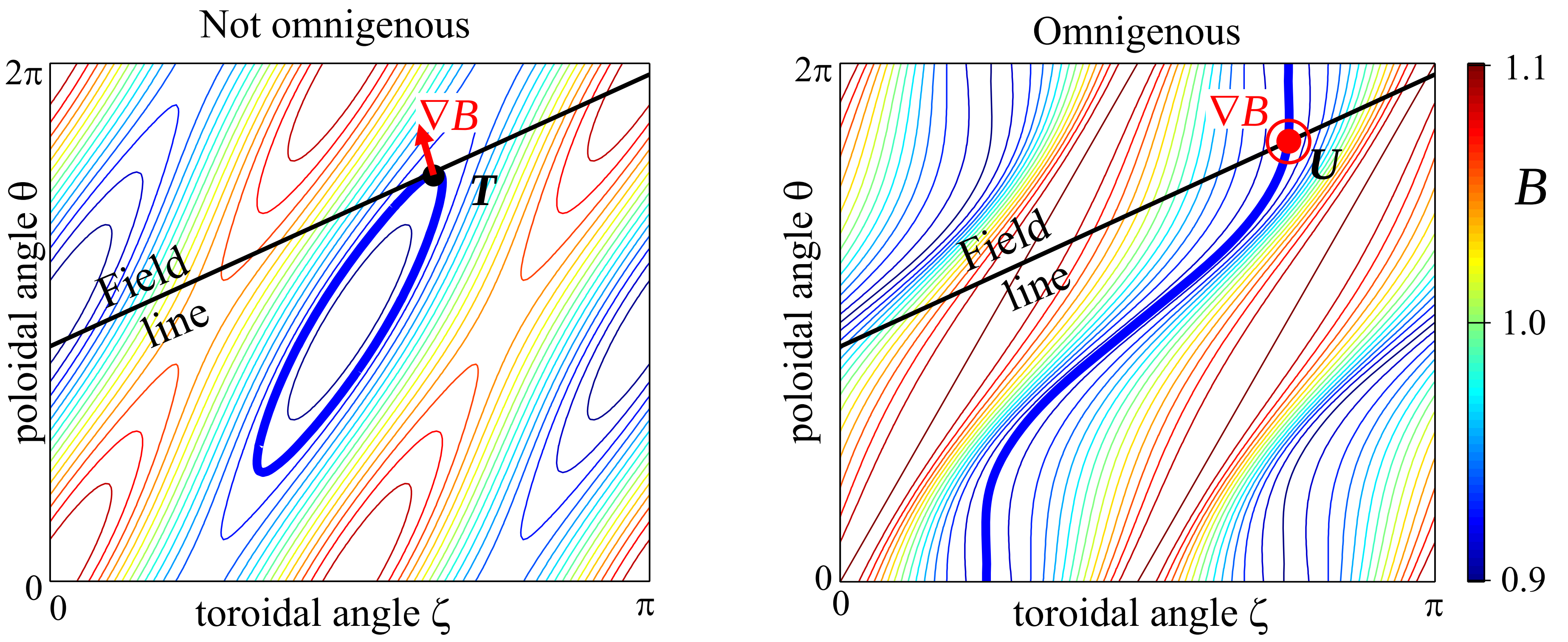

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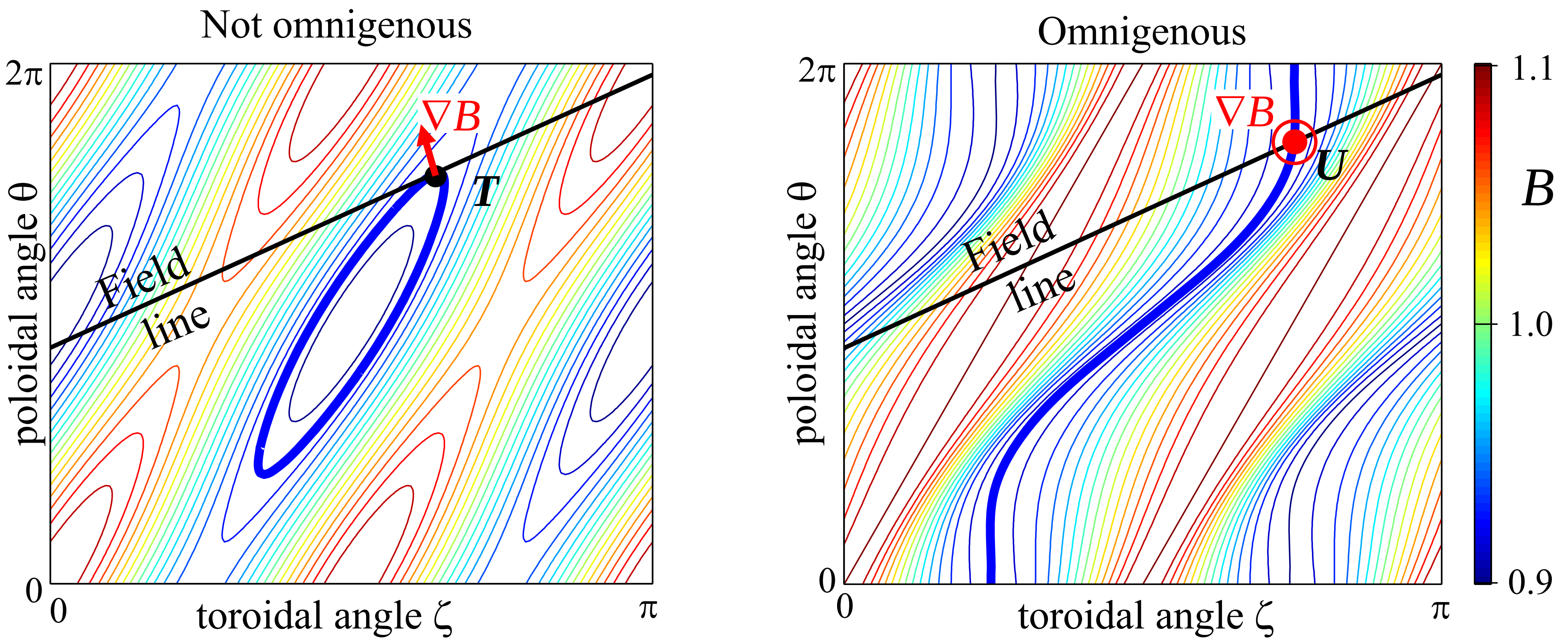

E.g., deeply trapped particles at T would see a nonzero $\mathbf{v}_d \cdot \nabla \psi \;\propto\; \mathbf{B} \times \nabla B \cdot \nabla \psi$

$\Rightarrow$ All B contours must link the torus toroidally, poloidally, or both.

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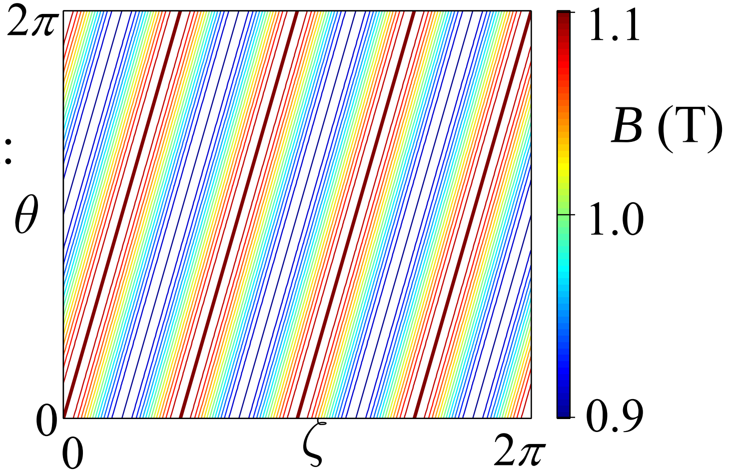

M = 1, N = 4

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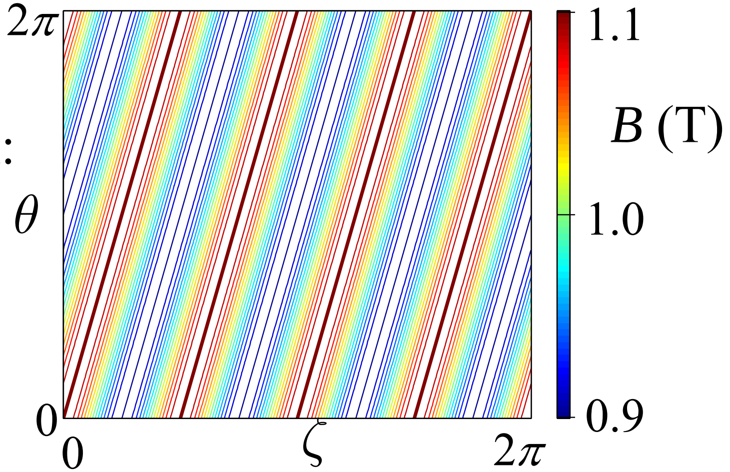

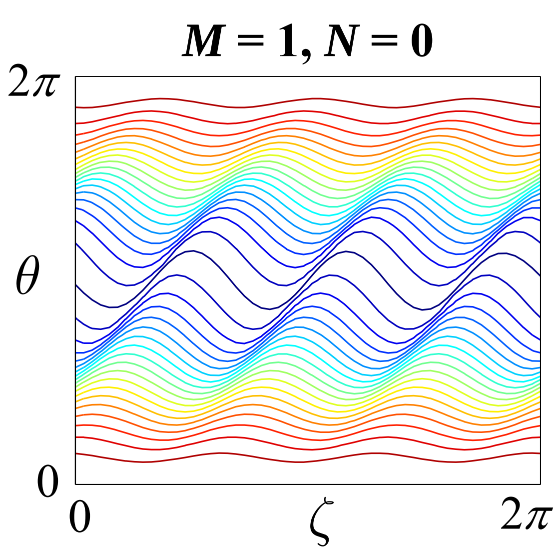

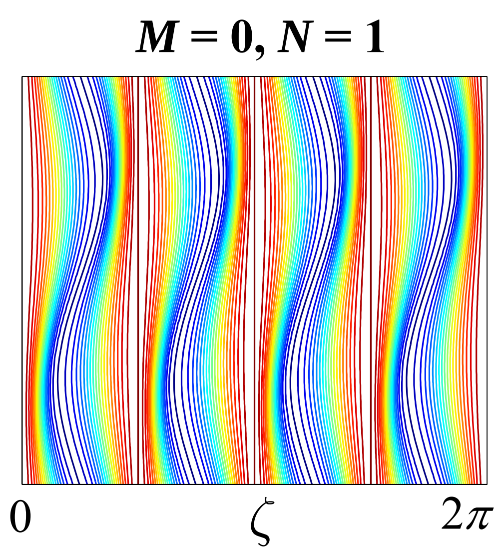

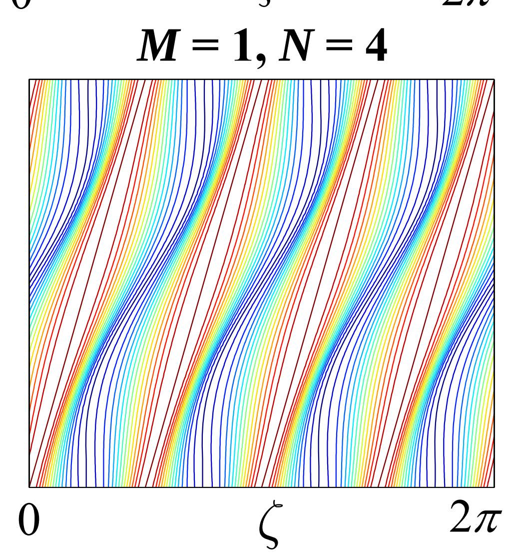

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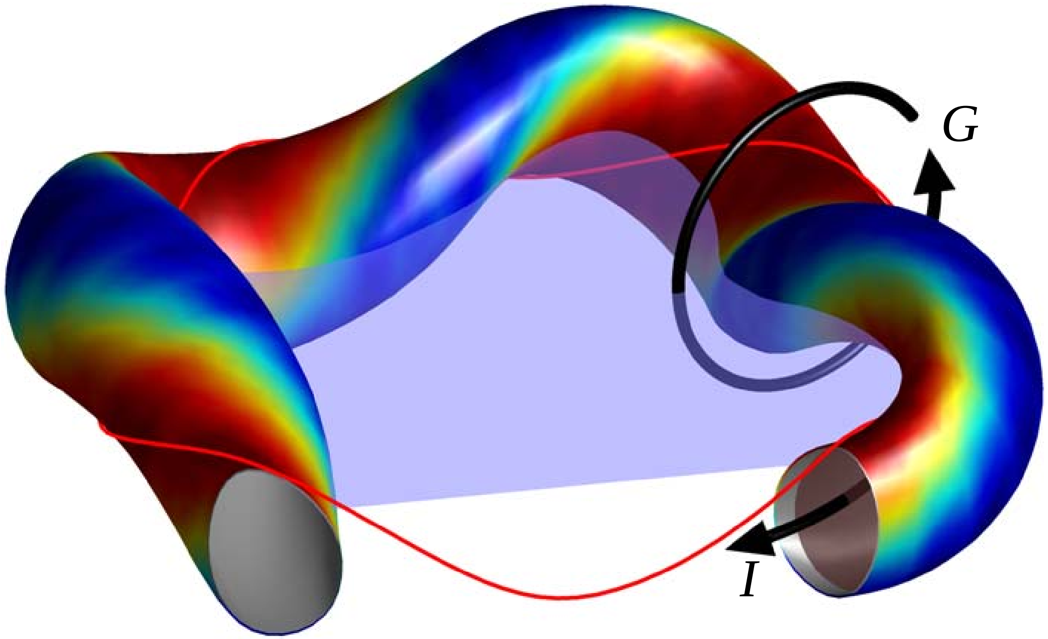
<u>New geometric consequence of omnigenity:</u>

Apply Ampère's Law to a B contour on a flux surface:

$$\oint \mathbf{B}\cdot d\mathbf{r} = \frac{4\pi}{c} \times \underbrace{(\text{enclosed current})}_{MG+NI}$$



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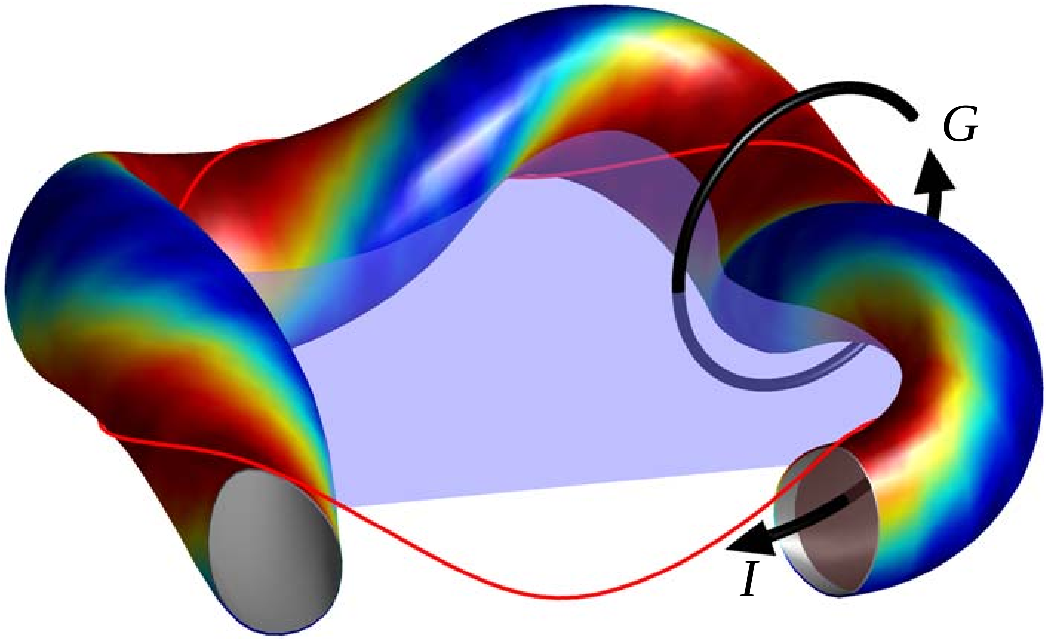
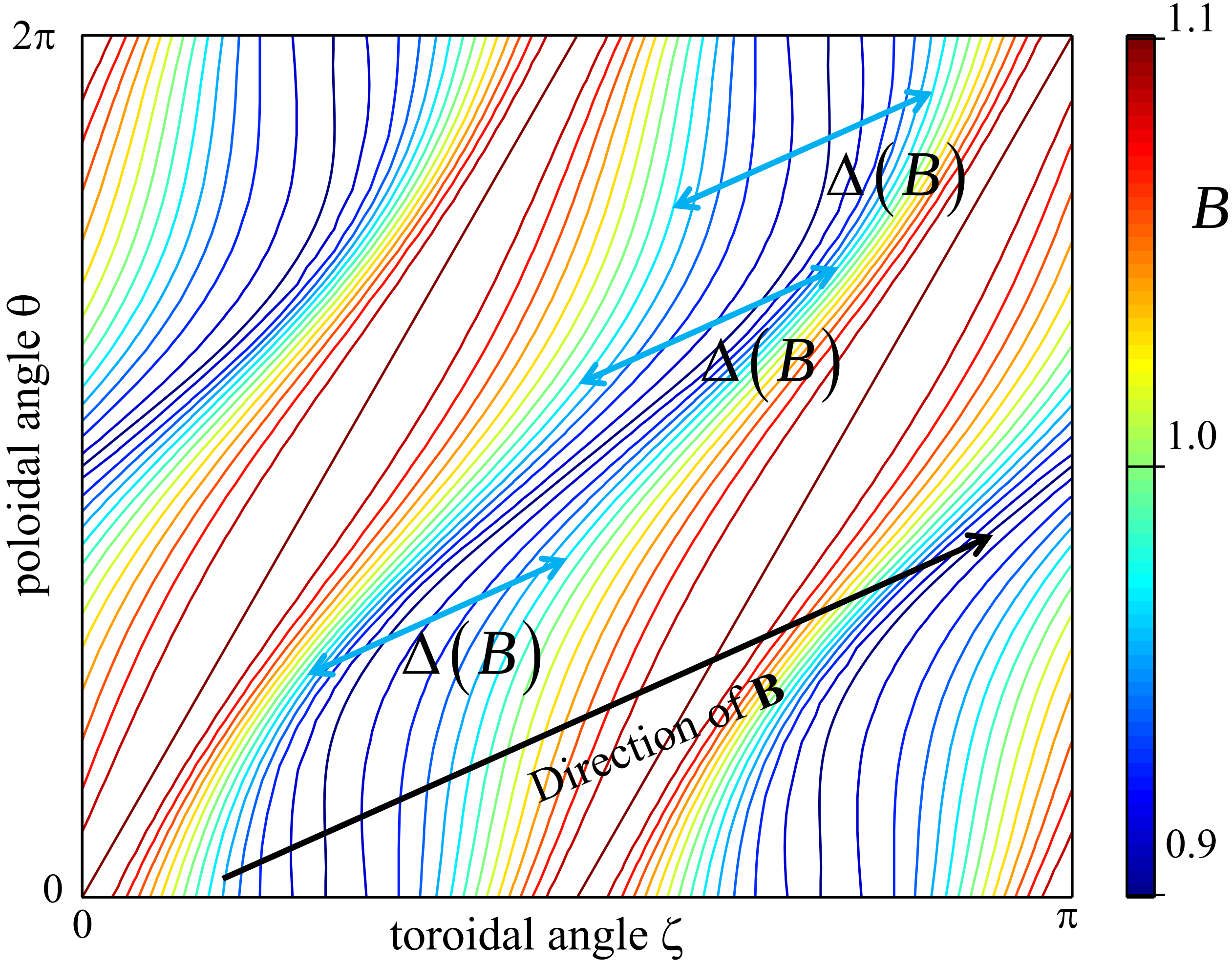


$$\Rightarrow \quad \frac{\mathbf{B}\times\nabla\psi\cdot\nabla B}{\mathbf{B}\cdot\nabla B} = \frac{2q}{c}\left(\frac{MG+NI+H}{M-qN}\right) \quad \text{where } \langle H\rangle = 0.$$

Omnigenous $B(\theta,\zeta)$ patterns can be constructed with a lot of freedom.

- Exploit the fact that $\dfrac{\partial \Delta(\theta, B)}{\partial \theta} = 0 \iff$ omnigenity.



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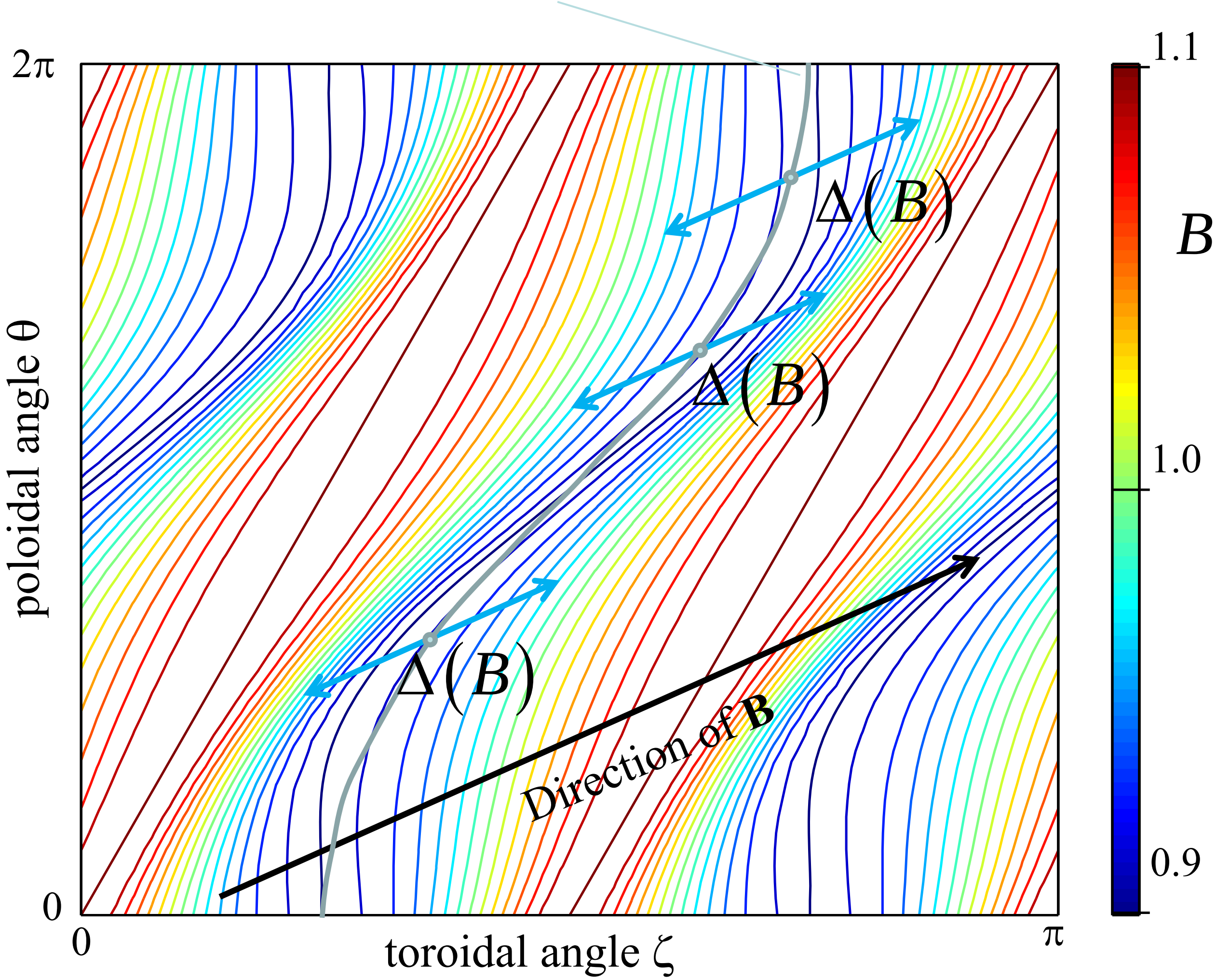
- Exploit the fact that $\dfrac{\partial \Delta(\theta, B)}{\partial \theta} = 0 \Leftrightarrow$ omnigenity.
- Choose any $\Delta(B)$ and $\zeta_0(\theta, B)$ (with constraints at B_{max} and B_{min}).



Can these optimized $B(\theta,\zeta)$ patterns be embedded in a real 3D equilibrium?

- Garren & Boozer, *Phys. Fluids B* **3**, 2822 (1991):
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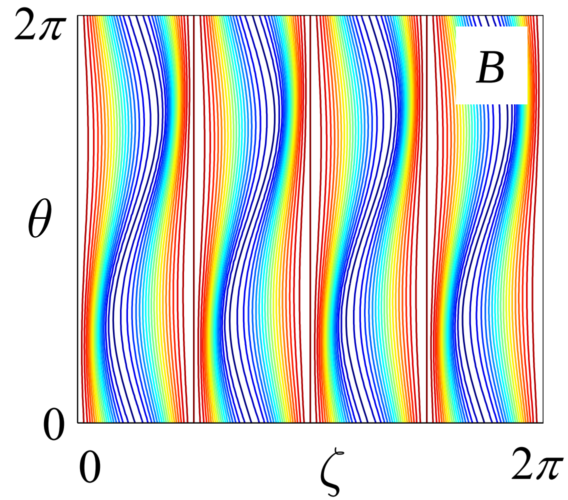
- Example: M=0 (generalized poloidal symmetry) is no longer prohibited:



$$\frac{\partial B}{\partial \theta}\neq 0 \text{ except at isolated points.}$$

Pressure-driven j and other properties of quasisymmetric plasmas are isomorphic to those in a tokamak.

Tokamak: $\left\langle j_{\|} B \right\rangle = -4.8\sqrt{\varepsilon} q \left(\frac{dp_i}{d\psi} + \frac{dp_e}{d\psi} - 0.74 n \frac{dT_e}{d\psi} - 1.17 n \frac{dT_i}{d\psi} \right) G$

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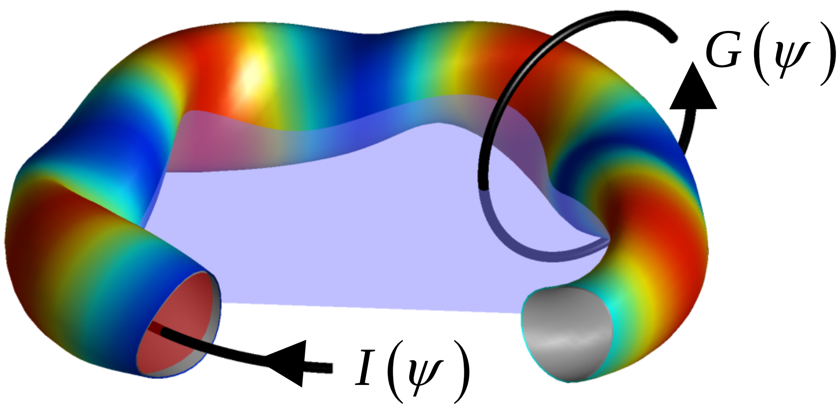
$B = B(\psi, M\theta - N\zeta)$

Pytte & Boozer PoF (1981), Boozer PoF (1983)

where $G(\psi)$ = poloidal current outside the flux surface,

$I(\psi)$ = toroidal current inside the flux surface



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General stellarator:

$$\left\langle j_{\|} B \right\rangle = -1.64 \frac{1}{f_c} \left[\left\langle g_2 \right\rangle - \frac{3\left\langle B^2 \right\rangle}{4 B_{\max}^2} \int_0^1 \frac{\left\langle g_4 \right\rangle}{\left\langle g_1 \right\rangle} \lambda \, d\lambda \right] \left[\frac{dp_i}{d\psi} + \frac{dp_e}{d\psi} - 0.74 n \frac{dT_e}{d\psi} - 1.17 n \frac{dT_i}{d\psi} \right]$$

where $g_1 = \sqrt{1 - \lambda B / B_{\max}}$, $\quad f_c = \frac{3\left\langle B^2 \right\rangle}{4 B_{\max}^2} \int_0^1 \frac{\lambda \, d\lambda}{\left\langle g_1 \right\rangle}$,

g_2 is defined by $\mathbf{B} \cdot \nabla \left(\frac{g_2}{B^2} \right) = \mathbf{B} \times \nabla \psi \cdot \nabla \left(\frac{1}{B^2} \right)$ and $g_2 = 0$ at $B = B_{\max}$,

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General stellarator: Less insightful, e.g. reverse of $\left\langle j_{\|} B \right\rangle$ in helical symmetry.

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Current in an omnigenous plasma is described by a concise, explicit, analytical formula.

$$j_{\|} = 3.3 \frac{f_t q B}{\langle B^2 \rangle} \left(\frac{NI + MG}{qN - M} \right) \left(\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} - 0.74 n_e \frac{dT_e}{d\psi} - 1.17 n_e \frac{dT_i}{d\psi} \right)$$

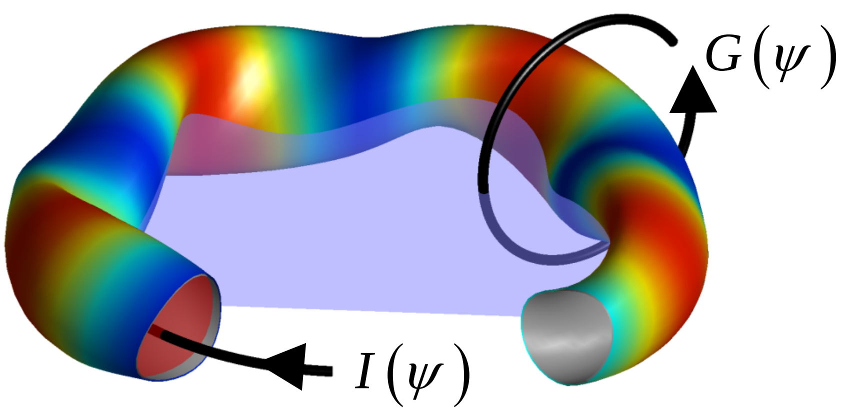
$$+ \frac{2q}{B(qN-M)} \left(\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} \right) \left[\left(1 - \frac{B^2}{\langle B^2 \rangle} \right) (NI + MG) + W \right]$$

Tokamak result with $G \to -(NI+MG)/(qN-M)$

$$W = \frac{2B^2}{q}(qG + I) \times \int^{\zeta} \frac{d\zeta'}{B'^3} \left(N \frac{\partial B'}{\partial \theta} + M \frac{\partial B'}{\partial \zeta} \right)$$

$$\langle WB \rangle = 0.$$

$I(\psi)$ and $G(\psi)$ are the toroidal & poloidal currents.



Flow in an omnigenous plasma is described by a concise, explicit, analytical formula.

$$V_{\parallel i} = -1.17 \frac{2qB}{e\langle B^2 \rangle} \frac{dT_i}{d\psi} \frac{(NI + MG)}{(qN - M)} + \frac{2q}{B}\left(\frac{d\Phi}{d\psi} + \frac{1}{en}\frac{dp_i}{d\psi}\right)\frac{(NI + MG + W)}{(qN - M)}$$

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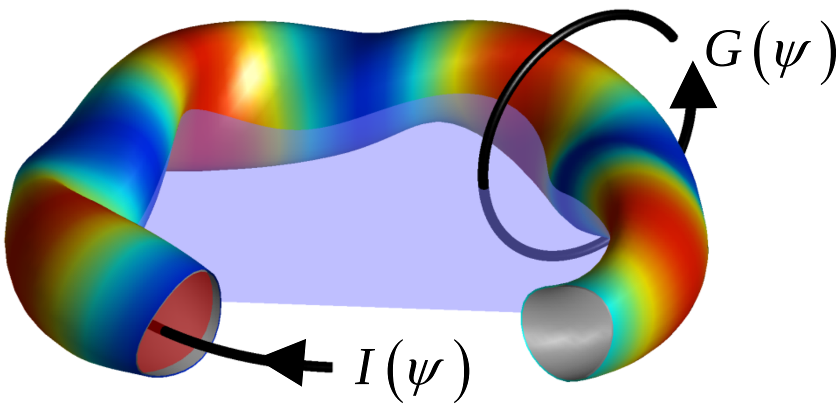

E_r in a perfectly quasisymmetric stellarator is determined differently than in a general stellarator.

Non-quasisymmetric stellarators:

- Neoclassical radial current depends on E_r.
- $\langle \mathbf{j}_{\text{neoclassical}} \cdot \nabla \psi \rangle \gg \langle \mathbf{j}_{\text{turbulence}} \cdot \nabla \psi \rangle$.

(*Helander & Simakov, Contrib. Plasma Phys. 2010*)

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Tokamaks & perfectly quasisymmetric stellarators:

- Neoclassical radial fluxes of ions and electrons are always equal, regardless of E_r ("intrinsic ambipolarity")

(*Helander & Simakov, PRL 2008*)

$\Rightarrow$ You cannot solve for E_r neoclassically. Turbulent $\mathbf{j}$ matters.

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Omnigenous stellarators: (new result)

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$$\frac{d\Phi}{d\psi} = \frac{1}{Ze} \left(-\frac{T_i}{n_i} \frac{dn_i}{d\psi} + 0.17 \frac{dT_i}{d\psi} \right)$$

Independent of the details of **B**.

Summary: omnigenity is an important limit.

- Relevant (at least for insight and code benchmarking) to any viable reactor.
- Easier to achieve than quasisymmetry, and α confinement and neoclassical transport are just as good.
- Using generalized helicity (M, N), concise, explicit, analytical formulae exist for f, $\mathbf{j}$, $\mathbf{V}$, and E_r.
- For omnigenous non-quasisymmetric $\mathbf{B}$, E_r is determined explicitly:

$$\frac{d\Phi}{d\psi} = \frac{1}{Ze}\left(-\frac{T_i}{n_i}\frac{dn_i}{d\psi} + 0.17\frac{dT_i}{d\psi} \right).$$

Landreman & Catto, Phys. Plasmas **19**, 056103 (2012)

Extra slides

Bootstrap j can vanish if $M = 0$.

Subbotin *et al*, NF **46**, 921 (2006), Helander & Nührenberg, PPCF **51**, 055004 (2009).

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So if B contours close poloidally ($M = 0$) rather than toroidally or helically,

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